

Constraints on the Average Density of Gravitationally Confined Objects with the Spin Period

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Abstract We propose to use the rotation period to constrain the average density of an object with gravitationally confined surface. The average density is inverse proportional to the square of the rotation period, while independent of the size of the object. The lower limit of the average density can be written as $\rho_0 = 10.9 \text{ g cm}^{-3} \left(\frac{\text{hours}}{P}\right)^2$. An asteroid with rotating period shorter than 0.7 h should consist of some unknown matter, or it is a whole rock or a bulk of ice with no rubble piles on the surface.

Key words: minor planets, asteroids: general, stars: rotation

1 INTRODUCTION

There is a lower limit on the spin period of a rotating object, which is determined by the equilibrium between the centrifugal force and the confining force. In the case of a pulsar, if it is consisted of neutron matter, the lower limit of its spin period is $\sim 1 \text{ ms}$ (Shapiro & Teukolsky 1983), resulting from the equilibrium between the centrifugal force and the gravitational force, since a neutron star is confined by gravitational force. A pulsar with sub-millisecond spin period would have different composition, which provide addition confinement, e.g. a quark star have confinement from the strong nuclear force. To find compact stars containing quark matter, one possible way is to search for sub-millisecond pulsars (Zheng et al. 2006).

In main sequence stars, the massive stars systematically rotate faster than the low mass stars. There have been some massive stars with critical rotating velocity found in the Milky Way (e.g., MWC 297, with spin period of $\sim 7 \text{ h}$, Acke et al. 2008). For comparison, the rotation period of the Sun is $\sim 25 \text{ d}$ (Kuiper 1953).

The average density of an object (e.g. an asteroid) is an important parameter to infer its contents. This parameter is important for both astrophysics and space mining. The average density can be used to constrain

the nature of the faint companion in a pulsar binary (e.g. in M71E, Pan et al. 2023). Asteroids rich in precious metals would have relatively higher density. Usually, the density is calculated with the mass and volume. In practice, it is difficult to measure the mass and radius of an object directly. The measurement of the average density is not easy. Here we demonstrate that it is possible to give a constraint to the average density with the spin period. In this work, we describe the method for constraining the average density in section 2. The data and results are presented in section 3. Then we give a further discussion in section 4. The conclusion is given in section 5

2 METHODS

For simplicity, consider a spherical object with radius r . At the edge of the equatorial plane, the acceleration due to centrifugal force is

$$a_c = \omega^2 r = 0.30 \left(\frac{\text{hours}}{P} \right)^2 \left(\frac{r}{\text{km}} \right) \text{ cm/s}^2, \quad (1)$$

where ω is the angular velocity, while the acceleration due to gravity is

$$a_g = \frac{4\pi}{3} \frac{G\rho_0 r^3}{r^2} = \frac{4\pi}{3} G\rho_0 r = 0.028 \left(\frac{\rho_0}{\text{g/cm}^3} \right) \left(\frac{r}{\text{km}} \right) \text{ cm/s}^2, \quad (2)$$

Where G is the gravitational constant, ρ_0 is the average density. The ratio of centrifugal acceleration to gravitational acceleration, f_{cg} can be written as

$$f_{cg} \equiv \frac{a_c}{a_g} = \frac{\omega^2 r}{\frac{4\pi}{3} G\rho_0 r} = \frac{3}{4\pi} \frac{\omega^2}{G\rho_0}. \quad (3)$$

So

$$\rho_0 = \left(\frac{3}{4\pi G} \right) \frac{\omega^2}{f_{cg}} = 10.9 \text{ g cm}^{-3} \left(\frac{\text{hours}}{P} \right)^2 \left(\frac{1}{f_{cg}} \right). \quad (4)$$

If an object is kept integrated by the gravitational force, the gravitational acceleration should be larger than centrifugal acceleration, $f_{cg} < 1$. When $f_{cg} = 1$, ρ_0 serves as a lower limit of the average density. On the other hand, the limiting period corresponding to ρ_0 is

$$P = 3.3 \text{ hours} \left(\frac{\text{g cm}^{-3}}{\rho_0} \right)^{1/2} \left(\frac{1}{f_{cg}} \right)^{1/2}. \quad (5)$$

It can be seen from Equation 2 that the average density of a gravitationally bound object is simply constrained by its spin period, while independent of its radius. To find candidates with high-density matter, a possible way is to find fast rotating objects.

3 DATA AND RESULTS

3.1 Spin Period Distribution

We collected the basic parameters of a sample of asteroids in Table ??, from 3D Asteroid Catalogue¹. Most asteroids in this sample have mean diameter, spin period and albedo.

The distribution of the spin period can be seen in Fig. 1. The distribution of asteroids with spin period longer than 6 hours is a power law. There are much less asteroids with spin period shorter than 6 hours. We calculated the lower limit of the average density of the asteroids by assuming they are gravitationally confined, as shown in Fig. 2. The constraint on the average density is not strong for current sample.

¹ <https://3d-asteroids.space/>

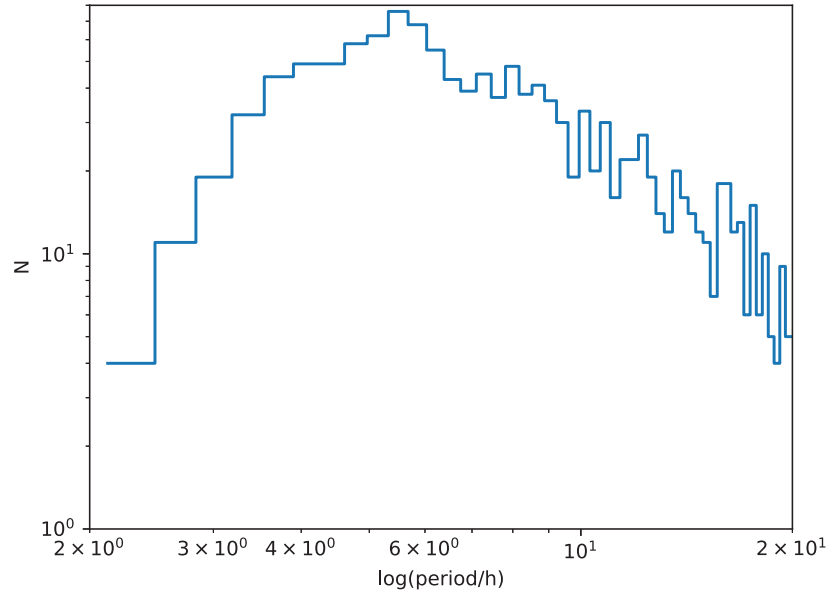


Fig. 1 The spin period distribution of the asteroids.

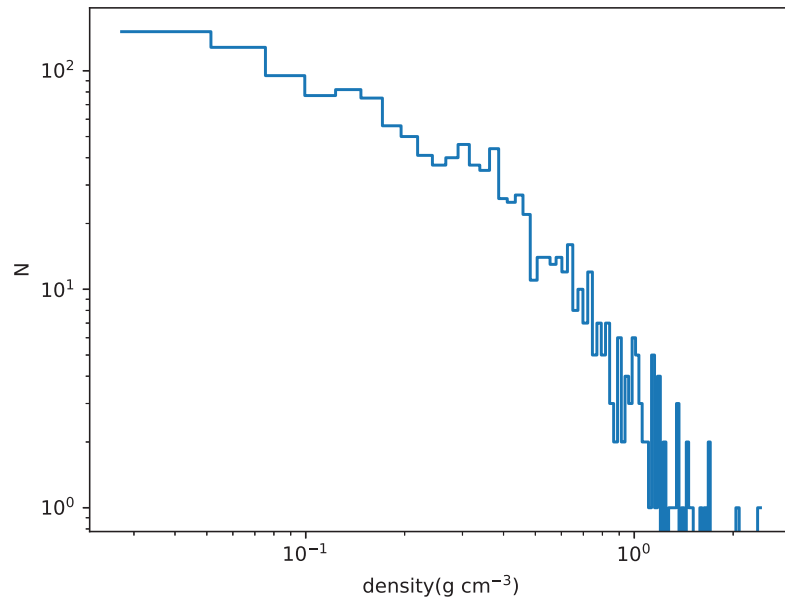


Fig. 2 The lower limits of the average density of the asteroids.

3.2 Asteroids With Density Measurements

There are 26 asteroids with additional parameters, e.g. mass, density and escape velocity (Table 1). We can compare these measurements with the density estimates from the spin periods. The estimated lower limit of the density is consistent with the measurements for most asteroids. For Asteroid (29075) 1950 DA, the esti-

mated lower limit of the density is larger than the density given by the catalogue. The detailed modelling of Asteroid (29075) 1950 DA shows that the average density should be 2.5 g cm^{-3} or 3.5 g cm^{-3} , consistent with our estimate.

Table 1 Physical Parameters of Asteroids with Density Measurements

Asteroids	Mean Diameter (km)	Spin period (hours)	Albedo	Mass (kg)	Density (g/cm^3)	Escape Velocity (km/s)	Estimated Density Lower limits (g/cm^3)
Asteroid (29075) 1950 DA	2.0000	2.1216	0.070	2×10^{12}	0.478	0.001	2.4
Asteroid (10115) 1992 SK	0.9380	7.3183	-	2.17×10^{12}	5.023	0.001	0.20
Asteroid (8567) 1996 HW1	2.9280	8.7624	-	2.27×10^{13}	1.727	0.001	0.14
Asteroid (52760) 1998 ML14	1.0000	14.280	-	1.09×10^{12}	2.082	0.001	0.05
Asteroid (33342) 1998 WT24	0.4320	3.6970	-	6.251×10^{11}	14.81	0.001	0.80
Asteroid (341843) 2008 EV5	0.4000	3.7250	-	3.431×10^{10}	1.024	-	0.79
Asteroid (101955) Bennu	0.4920	4.2975	0.046	7.793×10^{10}	1.250	-	0.59
Asteroid (107) Camilla	210.37	4.8439	0.059	1.12×10^{19}	2.298	0.119	0.46
Asteroid (45) Eugenia	202.33	5.6992	0.045	5.691×10^{18}	1.312	0.087	0.34
Asteroid (433) Eros	16.084	5.2702	0.250	6.689×10^{15}	2.675	0.010	0.39
Asteroid (15) Eunomia	231.69	6.0828	0.248	3.181×10^{19}	4.884	0.191	0.29
Asteroid (283) Emma	132.38	6.8952	0.023	1.38×10^{18}	1.136	0.053	0.23
Asteroid (10) Hygiea	407.12	27.659	0.072	1.049×10^{20}	2.969	0.262	0.01
Asteroid (121) Hermione	209.00	5.5509	0.048	4.71×10^{18}	0.0983	0.077	0.35
Asteroid (243) Ida	32.000	4.6336	0.262	4.121×10^{16}	2.402	0.019	0.51
Asteroid (704) Interamnia	306.31	8.7270	0.078	7.493×10^{19}	4.979	0.256	0.14
Asteroid (25143) Itokawa	0.3300	12.132	-	3.147×10^{10}	1.673	-	0.07
Asteroid (22) Kalliope	167.54	4.1482	0.166	7.358×10^{18}	2.989	0.108	0.63
Asteroid (216) Kleopatra	122.00	5.3853	0.116	4.641×10^{18}	4.881	0.101	0.38
Asteroid (253) Mathilde	52.800	417.70	0.044	1.033×10^{17}	1.340	0.023	6.3×10^{-5}
Asteroid (2) Pallas	545.00	7.8132	0.101	2.143×10^{20}	2.528	0.324	0.18
Asteroid (11) Parthenope	142.89	13.722	0.191	6.151×10^{18}	4.027	0.107	0.06
Asteroid (16) Psyche	226.00	4.1959	0.120	2.293×10^{19}	3.794	0.165	0.62
Asteroid (87) Sylvia	253.05	5.1836	0.046	1.48×10^{19}	1.745	0.125	0.41
Asteroid (17) Thetis	84.899	12.266	0.193	1.43×10^{18}	4.464	0.067	0.07
Asteroid (4) Vesta	525.40	5.3421	0.423	2.591×10^{20}	3.412	0.363	0.38

4 DISCUSSION

In our estimates above, We have assumed the asteroids are strengthless, totally confined by gravitational force. This is reasonable since space probes have taken images of some asteroids. It is shown that the surface is covered by rubble piles.

If the asteroids have confinement from other forces, their rotation period can be much shorter for the same average density. Some asteroids can consist of whole rock, or in other words, monolithic. For them, We can calculate the corresponding limiting spin period.

The typical tensile strength of a rock P_t is several million Pascal to several tens of million Pascal. For a cylinder of height $2h$ and bottom area A the centrifugal force would equal the breakup force at the breakup

limit. So the corresponding centrifugal acceleration is²

$$a_c = \frac{P_t A}{\rho h A} = \frac{P_t}{\rho h}, \quad (6)$$

where ρ is the density of the rock. The period at the breakup limits is

$$P = 62.8 \text{ s} \left(\frac{10 \text{ MPa}}{P_t} \right)^{1/2} \left(\frac{\rho}{1 \text{ g cm}^{-3}} \right)^{1/2} \left(\frac{h}{1 \text{ km}} \right). \quad (7)$$

The tensile strength of ice is 0.27 MPa. If an asteroid consists of a bulk of ice, the spin period of break up is then $P \sim 382 \text{ s}$.

Searching for extremely fast rotating objects will help find objects with high density and objects consisting of a single rock, or even exotic matter (eg. strangelets). This is important both for astrophysics and future asteroid mining, since most metallic minerals would have high density and form rock. Based on Eq. 2, an asteroid with rotating period of 0.5 h would have an average density of $\sim 40 \text{ g cm}^{-3}$, if it is gravitationally confined.

In common minerals, Iridium has the highest density of $\sim 20 \text{ g cm}^{-3}$ (Table A.1). It is believed that the iridium on the Earth comes from asteroids. If we find an asteroid with rotating period of $\sim 0.7 \text{ h}$, one possibility is that this asteroid consists of some unknown matter, since the density needed to provide enough gravitational confinement exceeds $\sim 20 \text{ g cm}^{-3}$. Another possibility is that it is a whole rock or a bulk of ice, so it is confined by lattice force rather than gravitational force. It is expected that there are no rubble piles on the surface of this asteroid.

In asteroids, the M-type asteroids contain more metal (mainly iron, nickel) (Tholen 1984). They have relatively higher average density. Observations also show that M-type asteroids rotate systematically faster than asteroids of other type (Belskaya & Lagerkvist 1996).

In the near Earth asteroids, there are fast-rotating asteroid. Asteroid 2001 OE84 has a period of 0.486 h (Pravec et al. 2002). Only objects smaller than about 200 m rotate more rapidly Warner et al. (2021). These asteroids must have an average density higher than $\sim 46 \text{ g cm}^{-3}$ if they are bounded by gravitational force. This density is too high for ordinary rock. Thus they are more likely to be consisted of a whole rock or a bulk of ice.

5 CONCLUSION

After considering the constraint of the density of a self-gravitating object from its rotation periods, we reach the following conclusion.

1. The rotation period provide a lower limit to the average density of an object, if its surface is gravitationally confined. This constraint does not depend on the radius of the object.
2. An object with spin period shorter than 0.7 hours could be consisting of high density matter or it consists of a single rock or a bulk of ice.
3. A fast rotating object with rubble pile on the surface should have a high average density.

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² If we assume the cylinder breakup from the middle.

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Appendix A: THE DENSITY OF METEORITES AND MINERALS WITH HIGH DENSITIES

For space mining on asteroids, we are mainly interested in precious metals, e.g. Iridium, Platinum. Minerals containing these elements are usually with relatively higher densities. We listed some minerals in Table A.1

Table A.1 The Density of Some Minerals

Minerals	Density (g cm ⁻³)
Silver	10.5
Sperrylite	10.58
Palladium	11.55
Gold	19.32
Iridosmine	19.3-21
Platinum	21.45
Iridium	22.7

- Data from The Engineering ToolBox (2009). Minerals - Densities. [online] Available at: https://www.engineeringtoolbox.com/mineral-density-d_1555.html [Accessed 9th July 2023].